#36 Measuring Distance 5.1

1. Some students have again decided to toss their calculus book. Below is a table that plots velocity in feet per second against time in seconds.

<table>
<thead>
<tr>
<th>$t=$ Time (sec)</th>
<th>0</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v =$ Velocity (ft/sec)</td>
<td>85.0</td>
<td>81.8</td>
<td>72.2</td>
<td>56.2</td>
<td>33.8</td>
<td>5.0</td>
</tr>
</tbody>
</table>

(a) How far has the car traveled? Since distance = velocity x time, we can approximate the distance traveled for each interval and then add the distances to get the total distance. First, graph the data from the chart:

![Graph of velocity vs. time](image)

Calculate the upper estimate:

Calculate the lower estimate:

Calculate the best estimate:

2. Calculate the following:
   (a) $(v_{upper} - v_{lower})(\text{length of the interval})$:

   (b) The difference between the upper estimate and the lower estimate:
Math 124

For a function that is increasing throughout or is decreasing throughout its interval \([a, b]\), the accuracy of its estimate depends on how close the lower estimate is to the upper estimate.

Let \( n \) = the number of measurements on \([a, b]\)
Let \( \Delta t \) = time interval between consecutive measurements
Error = \(|\text{upper estimate} - \text{lower estimate}| = |f(a) - f(b)| \cdot \Delta t

3. What is the error of our measurements in #1?
   How could we get a more accurate estimate?

4. Below is a chart in which measurements were taken more often:

<table>
<thead>
<tr>
<th>( t ) = Time (sec)</th>
<th>0</th>
<th>0.25</th>
<th>0.50</th>
<th>0.75</th>
<th>1.00</th>
<th>1.25</th>
<th>1.50</th>
<th>1.75</th>
<th>2.00</th>
<th>2.25</th>
<th>2.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v ) = Velocity (ft/sec)</td>
<td>85.0</td>
<td>84.2</td>
<td>81.8</td>
<td>77.8</td>
<td>72.2</td>
<td>65.0</td>
<td>56.2</td>
<td>45.8</td>
<td>33.8</td>
<td>20.2</td>
<td>5.00</td>
</tr>
</tbody>
</table>

Graph this data and then calculate the new and improved estimate.

5. Calculate how frequently must the velocity be recorded in order to estimate the total distance traveled to within 0.01 feet?